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# On the Exponentiated Generalized Weibull Distribution: A Generalization of the Weibull Distribution

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## Abstract

**Background/Objectives:** In this article, a generalization of the Weibull distribution is being studied in some details. The new model is referred to as the Exponentiated Generalized Weibull distribution. The aim is to increase the flexibility of the Weibull distribution. **Methods:** The concepts introduced in the Exponentiated Generalized family of distributions due to Cordeiro et al.[11] were employed. **Findings:** Some basic mathematical properties of the resulting model were identified and studied in minute details. Meanwhile, estimation of model parameters was performed using the maximum likelihood method. **Application/Improvement:** The Exponentiated Generalized Weibull distribution was presented as a competitive model that would be useful in modeling real life situations with inverted bathtub failure rates. The R-code for the plots was also provided. Further research would involve applying the proposed model to real life data sets.

**Keywords:** Exponentiated Generalized Weibull Distribution, Generalization, Inverted Bathtub Failure Rates, Weibull Distribution

## 1. Introduction

In probability distribution theory, the Weibull distribution is widely known as being versatile, relatively simple and it has received appreciable usage in the fields of engineering, medical sciences, weather forecasting, insurance and many more in recent times. It was identified by[1] and was named after[2].

In a bid to increase the flexibility of standard theoretical probability models, several notable authors have proposed various generalizations or generalized models; See[3–5]. Attempts to generalize the Weibull distribution include the work of[6] who introduced the Generalized Weibull (GW) distribution with a bathtub hazard (or failure) rate and the work of[7] who defined the Beta Weibull distribution. Many others are available in the literature.

According to [8], the works of[9] and[10] demonstrated the potentiality of the GW distribution in analyzing data sets relating to bus motor failure, head and neck cancer, and flood.

The Probability Density Function (pdf) and the Cumulative Density Function (cdf) of the GW distribution are given by;

$$f(x)=\alpha\beta\lambda^{\beta}x^{\beta-1}\left(1-e^{-(\lambda x)^{\beta}}\right)^{\alpha-1}e^{-(\lambda x)^{\beta}} \qquad (1)$$

For $x>0, \alpha>0, \beta>0, \lambda>0$

and $F(x)=\left(1-e^{-(\lambda x)^{\beta}}\right)^{\alpha}$ (2)

For $x>0, \alpha>0, \beta>0, \lambda>0$

respectively.

Where, $\alpha$ and $\beta$ are shape parameters.

$\lambda$ is a scale parameter

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On the contrary, this article intends to extend the Weibull distribution using the Exponentiated Generalized family of distribution due to[11]. In other words, we seek to extend the works of[7,11] with a view to defining and investigating a four parameter Exponentiated Generalized Weibull distribution in the same way the Inverse Exponential distribution was generalized in[12].

According to[11], the pdf and cdf of the Exponentiated Generalized (EG) class of distributions are given by;

$$f(x) = abg(x)\{1-G(x)\}^{a-1}\left[1-\{1-G(x)^a\}\right]^{b-1} \quad (3)$$

and

the corresponding cumulative distribution function is given by;

$$F(x) = \left[1-\{1-G(x)\}^a\right]^b \quad (4)$$

respectively.

Where, a,b > 0 are additional shape parameters.

G(x) is the cdf of the baseline (or parent) distribution and $g(x) = \frac{dG(x)}{dx}$

Equation (4) is easier to handle and relatively simpler than the Beta-G family of distributions due to[13], this is due to the fact that, Equation (4) does not include special functions like the incomplete beta function. Also, Equation (4) has tractability advantage for simulation purposes because its quantile function takes a simple form[11,12]. For detailed information on the physical interpretation of Equation (4), readers are referred to[11].

The rest of this article is structured as follows; in Section 2, the Exponentiated Generalized Weibull (EGWeibull) distribution is defined, in Section 3, expressions for some basic statistical properties of the model are provided, procedure for estimating the model parameters is given in Section 4, followed by a concluding remark. The R-code used for the plots in this research is provided as APPENDIX.

## 2. The Exponentiated Generalized Weibull Distribution

The pdf and cdf of the Weibull distribution with parameters α and β are given by;

$$g(x) = \frac{\alpha}{\beta}\left(\frac{x}{\beta}\right)^{\alpha-1} e^{-(x/\beta)^\alpha}; \quad x>0, \alpha>0, \beta>0 \quad (5)$$

and

$$G(x) = 1-e^{-(x/\beta)^\alpha}; \quad x>0, \alpha>0, \beta>0 \quad (6)$$

respectively.

Where, α is the shape parameter.

β is the scale parameter.

Also, the mean is given by;

$$E[X] = \beta\Gamma\left(1+\tfrac{1}{\alpha}\right) \quad (7)$$

$$Median = \beta\{\ln(2)\}^{1/\alpha} \quad (8)$$

and;

$$Var[X] = \beta^2\left[\Gamma\left(1+\frac{2}{\alpha}\right)-\left\{\Gamma\left(1+\frac{1}{\alpha}\right)\right\}^2\right] \quad (9)$$

Following the contents of[11,12], the Exponentiated Generalized Weibull (EGWeibull) distribution is derived by substituting Equations (5) and (6) into Equation (3). In a more clear term, if a random variable X is such that; $X \sim EGWeibull(a,b,\alpha,\beta)$, then its pdf is given by;

$$f(x) = ab\frac{\alpha}{\beta}\left(\frac{x}{\beta}\right)^{\alpha-1} e^{-(x/\beta)^\alpha}\left\{1-\left(1-e^{-(x/\beta)^\alpha}\right)\right\}^{a-1} \left[1-\left\{1-\left(1-e^{-(x/\beta)^\alpha}\right)^a\right\}\right]^{b-1} \quad (10)$$

For $x>0, a>0, b>0, \alpha>0, \beta>0$

The expression in Equation (10) can be reduced to;

$$f(x) = ab\frac{\alpha}{\beta}\left(\frac{x}{\beta}\right)^{\alpha-1} e^{-(x/\beta)^\alpha}\left(e^{-(x/\beta)^\alpha}\right)^{a-1}\left[\left(1-e^{-(x/\beta)^\alpha}\right)^a\right]^{b-1}$$

Therfore;

$$f(x) = ab\frac{\alpha}{\beta}\left(\frac{x}{\beta}\right)^{\alpha-1}\left(e^{-(x/\beta)^\alpha}\right)^a\left[\left(1-e^{-(x/\beta)^\alpha}\right)^a\right]^{b-1} \quad (11)$$

For

$x>0, a>0, b>0, \alpha>0, \beta>0$

Where a,b and α are shape parameters.

β is the scale parameter.

The associated cdf of the EGWeibull distribution is given by;

$$F(x)=\left[1-\left\{1-\left(1-e^{-(x/\beta)^{\alpha}}\right)\right\}^{a}\right]^{b} \quad x>0,a>0,b>0,\alpha>0,\beta>0 \quad (12)$$

The expression in Equation (12) is simplified to give;

$$F(x)=\left[1-\left(e^{-(x/\beta)^{\alpha}}\right)^{a}\right]^{b} \quad x>0,a>0,b>0,\alpha>0,\beta>0 \quad (13)$$

## 2.1 Expansions for the CDF

For any real non-integer, 'b' in[11], considered a power series expansion given by;

$$\left(1-z\right)^{b-1}=\sum_{k=0}^{\infty}\frac{(-1)^{k}\Gamma(b)}{\Gamma(b-k)k!}z^{k} \quad (14)$$

It is good to note that the expression in Equation (14) is valid for $|z|<1$. Using the binomial expansion for a positive real power, the resulting cdf is given by;

$$F(x)=\sum_{j=0}^{\infty}w_{j}G(x)^{j} \quad (15)$$

The coefficients $w_j = w_j(a,b)$ were given by;

$$w_{j}=\sum_{k=0}^{\infty}\frac{(-1)^{k+j}\Gamma(b+1)\Gamma(ak+1)}{\Gamma(b-k)k!j!}$$

With this knowledge, the cdf of the EGWeibull distribution can be re-written as;

$$F(x)=\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\frac{(-1)^{k+j}\Gamma(b+1)\Gamma(ak+1)}{\Gamma(b-k)k!j!}\left[1-\left\{e^{-(x/\beta)^{\alpha}}\right\}^{a}\right]^{bj} \quad (16)$$

It is good to note that the expression in Equation (16) is an infinite power series of the Weibull distribution.

With respect to the series expansion in Equation (14), author in[11] gave the pdf of the Exponentiated Generalized (EG) class of distributions (for 'a' real non-integer) as;

$$f(x)=abg(x)\sum_{j=0}^{\infty}t_{j}G(x)^{j} \quad (17)$$

The coefficients $t_j = t_j(a,b)$ were

$$t_{j}=\frac{(-1)^{j}\Gamma(b)}{j!}\sum_{k=0}^{\infty}\frac{(-1)^{k}\Gamma(a(k+1))}{\Gamma(b-k)\Gamma((k+1)a-j)k!} \quad (18)$$

Therefore, we re-write the pdf of the EGWeibull distribution as;

$$f(x)=ab\frac{\alpha}{\beta}\left(\frac{x}{\beta}\right)^{\alpha-1}e^{-(x/\beta)^{\alpha}}\sum_{j=0}^{\infty}\frac{(-1)^{j}\Gamma(b)}{j!}\sum_{k=0}^{\infty}\frac{(-1)^{k}\Gamma(a(k+1))}{\Gamma(b-k)\Gamma((k+1)a-j)k!}\left[1-\left\{e^{-(x/\beta)^{\alpha}}\right\}^{a}\right]^{bj} \quad (19)$$

## 2.2 Relationship with Other Distributions

Some important models in the literature are special cases of the EGWeibull distribution. For example,

- For a=1, Equation (11) reduces to give the Generalized Weibull (GW) distribution.
- For b=1, Equation (11) reduces to give the ExponentiatedWeibull (EW) distribution.
- For a=b=1, Equation (11) reduces to give the Weibull distribution (which is the baseline distribution).
- For a=b= α =1, Equation (11) reduces to give the Exponential distribution.

For brevity, a plot for the pdf of the EGWeibull distribution at a=2, b=2, α =2, β =3, is given in Figure 1;

The plot as shown in Figure 1 indicates that the shape of the EGWeibull distribution could be unimodal. This would be further confirmed in the next section.

A possible plot for the cdf of the EGWeibull distribution at a=2, b=2, α =2, β =3, is given in Figure 2;

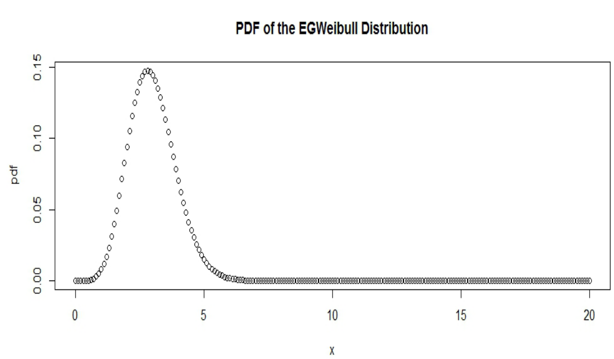


**Figure 1.** Plot for the pdf of the EGWeibull distribution a=2, b=2, α =2, β =3 .

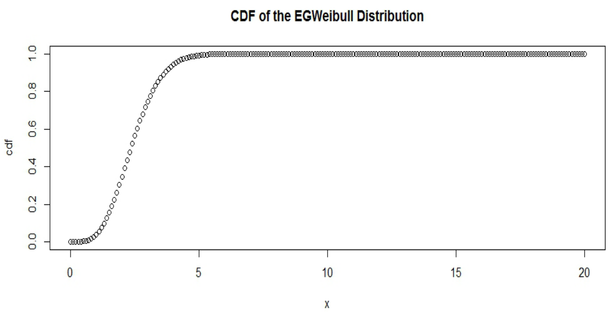


**Figure 2.** Plot for the cdf of the EGWeibull distribution a=2, b=2, α =2, β =3.

# 3. Properties of the EGWeibull Distribution

Some basic statistical properties of the EGWeibull distribution are identified in this section as follows

## 3.1 Limiting Behavior

The behavior of the pdf of EGWeibull distribution is being investigated as $x\to0$ and as $x\to\infty$. That is, $\lim_{x\to0} f(x)$ and $\lim_{x\to\infty} f(x)$ are considered

As $x\to0$;

$$\lim_{x\to0} f(x)=\lim_{x\to0}\left[ab\frac{\alpha}{\beta}\left(\frac{x}{\beta}\right)^{\alpha-1} e^{-(x/\beta)^{\alpha}}\left(e^{-(x/\beta)^{\alpha}}\right)^{a-1}\left[\left(1-e^{-(x/\beta)^{\alpha}}\right)^{a}\right]^{b-1}\right]$$

$$=0$$

Note: This is because; as $x\to0$, the expression $1-e^{-(x/\beta)^{\alpha}}$ becomes zero.

As $x\to\infty$;

$$\lim_{x\to\infty} f(x)=\lim_{x\to\infty}\left[ab\frac{\alpha}{\beta}\left(\frac{x}{\beta}\right)^{\alpha-1} e^{-(x/\beta)^{\alpha}}\left(e^{-(x/\beta)^{\alpha}}\right)^{a-1}\left[\left(1-e^{-(x/\beta)^{\alpha}}\right)^{a}\right]^{b-1}\right]$$

$$=0$$

These results affirm that the proposed EGWeibull distribution has a unique mode.

In the same way, the behavior of the cdf of EGWeibull distribution as given in Equation (13) is being investigated as $x\to0$ and as $x\to\infty$. That is, $\lim_{x\to0} F(x)$ and $\lim_{x\to\infty} F(x)$ are considered.

As $x\to0$;

$$\lim_{x\to\infty} F(x)=\lim_{x\to\infty}\left\{\left[1-\left(e^{-(x/\beta)^{\alpha}}\right)^{a}\right]^{b}\right\}$$

$$=0$$

As $x\to\infty$;

$$\lim_{x\to\infty} F(x)=\lim_{x\to\infty}\left\{\left[1-\left(e^{-(x/\beta)^{\alpha}}\right)^{a}\right]^{b}\right\}$$

$$=1$$

## 3.2 Reliability Analysis

Here, expressions for the survivor (or reliability) function and the hazard rate of the EGWeibull distribution are provided.

Mathematically, the reliability function is given by;

$$S(x)=P(\{X>x\})=\int_x^{\infty} f(u)\,du=1-F(x)$$

Therefore, the survivor function of the EGWeibull distribution is expressed as;

$$S_{EGWeibull}(x)=1-\left[1-\left\{e^{-(x/\beta)^{\alpha}}\right\}^{a}\right]^{b} \tag{20}$$

For $x>0, a>0, b>0, \alpha>0, \beta>0$

A plot for the survivor function of the EGWeibull distribution is given in Figure 3;

Mathematically, hazard (or failure) rate is given by;

$$h(x)=\frac{f(x)}{1-F(x)}$$

Therefore the hazard rate for the EGWeibull distribution is given by;

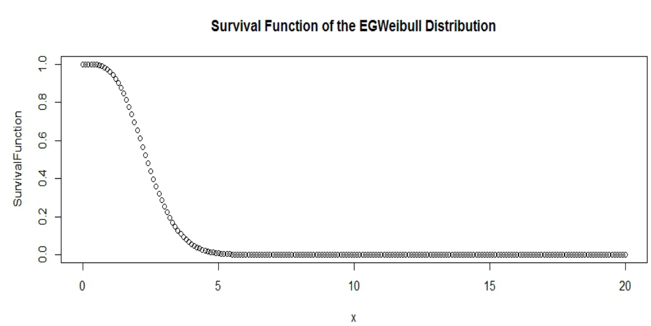


**Figure 3.** Plot for the survivor function of the EGWeibull distribution a=2, b=2, α =2, β =3.

$$h(x)=\frac{ab\frac{\alpha}{\beta}\left(\frac{x}{\beta}\right)^{\alpha-1}\left(e^{-(x/\beta)^{\alpha}}\right)^{a}\left[\left(1-e^{-(x/\beta)^{\alpha}}\right)^{a}\right]^{b-1}}{1-\left[1-\left\{e^{-(x/\beta)^{\alpha}}\right\}^{a}\right]^{b}} \tag{21}$$

For $x>0, a>0, b>0, \alpha>0, \beta>0$

A plot for the hazard function at a=2, b=2, α =2, β =3 is as shown in Figure 4;

The plot in Figure 4 shows that the EGWeibull distribution has an inverted bathtub failure rate. This implies that the EGWeibull distribution can be used to model real life situations with inverted bathtub failure rates.

## 3.3 Moments

According to[11], the moments of any EG distribution can be expressed as an infinite weighted sum of probability weighted moments of the parent distribution.

$$E\left[X^{r}\right]=a\beta\sum_{j=0}^{\infty}t_{j}\tau_{rj} \tag{22}$$

$\tau_{rj}$ in this case is based on the quantile function of the Weibull distribution; $Q_G(u)=G^{-1}(x)$.

Following[11], let $G(x)=u$;

$$\tau_{rj}=\int_{0}^{1}Q_G(u)^{r}u^{j}du$$

$t_j$ is as defined in Equation (18).

In particular, the quantile function for the Weibull distribution is given by;

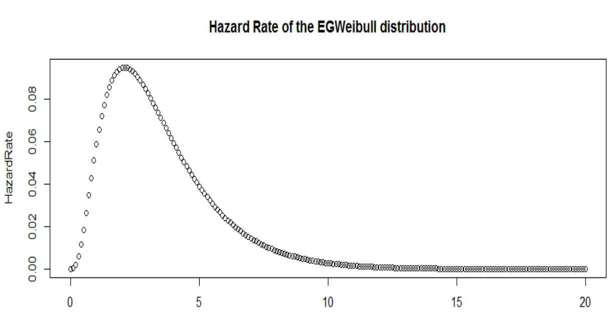


**Figure 4.** Plot for the hazard rate of the EGWeibull distribution a=2, b=2, α =2, β =3.

$$Q_G(u)=-\beta\left[\ln(1-u)\right]^{1/\alpha} \tag{23}$$

Therefore;

$$\tau_{rj}=(-1)^{r}\beta^{r}\int_{0}^{1}u^{j}\left\{\left[\ln(1-u)\right]^{1/\alpha}\right\}^{r}du \tag{24}$$

Inserting Equations (18) and (24) into Equation (22) gives the expression for the rth moment of the EGWeibull distribution.

## 3.4 Quantile Function

The quantile function Q has been identified as an alternative to the pdf as it is a way of prescribing a probability distribution. It is derived as the inverse of the cdf. With this understanding, the quantile function of the EGWeibull distribution can be expressed as;

$$Q(u)=-\beta\left[\ln\left(1-u^{1/b}\right)^{1/a}\right]^{1/\alpha} \tag{25}$$

Substituting $u = 0.5$ gives the median. Therefore, the median is given by;

$$Q_{0.5}=-\beta\left[\ln\left(1-(0.5)^{1/b}\right)^{1/a}\right]^{1/\alpha} \tag{26}$$

Random variables from the EGWeibulldistribution can be gotten using the expression;

$$X=-\beta\left[\ln\left(1-u^{1/b}\right)^{1/a}\right]^{1/\alpha} \tag{27}$$

## 3.5 Order Statistics

The pdf of the *ith* order statistic for *i=1,2.....,n* from independently and identically distributed random variables $X_1, X_2, \ldots, X_2$ is given by;

$$f_{i:n}(x)=\frac{f(x)}{B(i,n-i+1)}F(x)^{i-1}\left\{1-F(x)\right\}^{n-i} \tag{28}$$

Following[11], the distribution of order statistics for the EG family of distributions is expressed as;

$$f_{i:n}(x)=\frac{ab}{B(i,n-i)}g(x)\left\{1-G(x)\right\}^{a-1}\times\left[1-\left\{1-G(x)\right\}^{a}\right]^{bi-1}\times\left\{1-\left[1-\left\{1-G(x)\right\}^{a}\right]^{b}\right\}^{n-i} \tag{29}$$

With the aid of binomial expansion, the distribution of order statistics for the EGWeibull distribution can be written as;

$$f_{i:n}(x)=\frac{ab}{B(i,n-i)}\frac{\alpha}{\beta}\left(\frac{x}{\beta}\right)^{\alpha-1}e^{-(x/\beta)^{\alpha}}\left\{1-\left(1-e^{-(x/\beta)^{\alpha}}\right)\right\}^{a-1}$$
$$\times\sum_{k=0}^{n-i}(-1)^{k}\binom{n-i}{k}\times\left[1-\left\{1-\left(1-e^{-(x/\beta)^{\alpha}}\right)\right\}^{a}\right]^{b(i+k)-1} \quad (30)$$

The expression in Equation (31) can be simplified to give;

$$f_{i:n}(x)=\frac{ab}{B(i,n-i)}\frac{\alpha}{\beta}\left(\frac{x}{\beta}\right)^{\alpha-1}e^{-(x/\beta)^{\alpha}}\left(e^{-(x/\beta)^{\alpha}}\right)^{a-1}$$
$$\times\sum_{k=0}^{n-i}(-1)^{k}\binom{n-i}{k}\times\left[1-\left(e^{-(x/\beta)^{\alpha}}\right)^{a}\right]^{b(i+k)-1} \quad (31)$$

## 4. Estimation

The parameters of the EGWeibull distribution can be estimated using the method of maximum likelihood. Let $X_1, X_2,.....,X_n$ denote a random sample of size $n$ from the *EGWeibull* (a, b, α, β) distribution. The likelihood function is given by;

$$L\left(\underset{\sim}{X}\,|\,a,b,\alpha,\beta\right)=\prod_{i=1}^{n}\left[ab\frac{\alpha}{\beta}\left(\frac{x}{\beta}\right)^{\alpha-1}\left(e^{-(x/\beta)^{\alpha}}\right)^{a}\left[\left(1-e^{-(x/\beta)^{\alpha}}\right)^{a}\right]^{b-1}\right] \quad (32)$$

Define $l=\log L\left(\underset{\sim}{X}\,|\,a,b,\alpha,\beta\right)$

Therefore, the log-likelihood function is given by;

$$l=n\log a+n\log b+n\log\alpha-n\log\beta+(\alpha-1)$$
$$\sum_{i=1}^{n}\log\left(\frac{x_i}{\beta}\right)-a\sum_{i=1}^{n}\left(\frac{x_i}{\beta}\right)^{\alpha}+a(b-1)\sum_{i=1}^{n}\log\left(1-e^{-\left(x_i/\beta\right)^{\alpha}}\right) \quad (33)$$

The solutions of $\frac{\partial l}{\partial a}=0,\frac{\partial l}{\partial b}=0,\frac{\partial l}{\partial\alpha}=0$ and $\frac{\partial l}{\partial\beta}=0$ gives the maximum likelihood estimate for parameters a, b, α, and β respectively.

The solution of the non-linear systems of equations may not be easily derived analytically but can be gotten numerically using statistical software like SAS or R.

## 5. Conclusion

This article studies a four parameter Exponentiated Generalized Weibull distribution by generalizing the two-parameter weibull distribution. The aim is to induce skewness into the parent distribution in order to increase its flexibility and capability to model data sets that are more skewed. The model is unimodal and it has the Generalized Weibull distribution, Exponentiated Weibull distribution, Weibull distribution and the Exponential distribution as sub models. Some basic properties of the EGWeibull distribution are identified and the model is capable of modeling real life situations with inverted bathtub shape. Further research would involve an application of the EGWeibull distribution to a real data set to assess its flexibility over its sub models.

## APPENDIX

R-code for the plots;

```
> x=seq(0,20,0.1)
>x
 [1] 0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0 1.1 1.2 1.3 1.4
1.5 1.6 1.7 1.8 1.9 2.0 2.1 2.2
 [24] 2.3 2.4 2.5 2.6 2.7 2.8 2.9 3.0 3.1 3.2 3.3 3.4 3.5 3.6 3.7
3.8 3.9 4.0 4.1 4.2 4.3 4.4 4.5
 [47] 4.6 4.7 4.8 4.9 5.0 5.1 5.2 5.3 5.4 5.5 5.6 5.7 5.8 5.9 6.0
6.1 6.2 6.3 6.4 6.5 6.6 6.7 6.8
 [70] 6.9 7.0 7.1 7.2 7.3 7.4 7.5 7.6 7.7 7.8 7.9 8.0 8.1 8.2 8.3
8.4 8.5 8.6 8.7 8.8 8.9 9.0 9.1
 [93] 9.2 9.3 9.4 9.5 9.6 9.7 9.8 9.9 10.0 10.1 10.2 10.3 10.4
10.5 10.6 10.7 10.8 10.9 11.0 11.1 11.2 11.3 11.4
[116] 11.5 11.6 11.7 11.8 11.9 12.0 12.1 12.2 12.3 12.4 12.5
12.6 12.7 12.8 12.9 13.0 13.1 13.2 13.3 13.4 13.5 13.6 13.7
[139] 13.8 13.9 14.0 14.1 14.2 14.3 14.4 14.5 14.6 14.7 14.8
14.9 15.0 15.1 15.2 15.3 15.4 15.5 15.6 15.7 15.8 15.9 16.0
[162] 16.1 16.2 16.3 16.4 16.5 16.6 16.7 16.8 16.9 17.0 17.1
17.2 17.3 17.4 17.5 17.6 17.7 17.8 17.9 18.0 18.1 18.2 18.3
[185] 18.4 18.5 18.6 18.7 18.8 18.9 19.0 19.1 19.2 19.3 19.4
19.5 19.6 19.7 19.8 19.9 20.0
> a=2
> b=2
> c=2
> d=3
>    pdf=a*b*(c/d)*(x/d)^(c-1)*(exp(-(x/d)^c))^a*((1-
exp(-(x/d)^c))^a)^(b-1)
>plot(x,pdf,main="PDF of the EGWeibull Distribution")
> a=2
> b=2
```

```
> c=2
> d=3
>cdf=(1-(exp(-(x/d)^c))^a)^b
>plot(x,cdf,main="CDF of the EGWeibull Distribution")
>SurvivalFunction=1-cdf
>plot(x,SurvivalFunction,main="Survival Function of the EGWeibull Distribution")
>HazardRate=pdf/SurvivalFunction
>plot(x,HazardRate,main="Hazard Rate of the EGWeibull Distribution")
```

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